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SPECIAL BRIEF NOTE

Dedicated to Dr DAVID J. MAULL

ASPIRATING PIPES DO NOT FLUTTER AT INFINITESIMALLY SMALL FLOW

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In earlier theoretical work it has been found that cantilevered pipes aspirating fluid at their free end and conveying it toward the clamped end lose stability by flutter at infinitesimally small flow velocities. It is shown here that this is false, and the necessary theoretical correction is given. Moreover, an experiment is described supporting this new finding: aspirating pipes appear to remain stable to at least high flow rates.

1. INTRODUCTION

EVER SINCE Païdoussis & Luu's (1985) work, it has tacitly been accepted that cantilevered pipes aspirating fluid (i.e., with the fluid entering the free end and flowing towards the clamped one) lose stability by flutter at infinitesimally small flow velocities. It should be mentioned that this theoretical finding was never confirmed by experiment, as pointed out by Dupuis & Rousselet (1991) and Païdoussis (1991). In fact, it has recently been shown that this theoretical result is totally false (Païdoussis 1998), but unfortunately not soon enough to stop others from following the earlier, false theory, as a minor or major component of their work.

Whether the system loses stability at infinitesimally small flow is important not only in terms of fundamentals, but also in practical engineering terms, namely, in the field of Ocean Mining.

It was therefore thought to be useful to publicize the new finding in a separate paper, as a service to the research community. The epigrammatic title follows the venerable and laudable tradition instituted by Holmes (1978) in which the main conclusion of a paper is succinctly stated in the title—in these busy times, a practice of considerable appeal.

2. BACKGROUND AND HISTORICAL PERSPECTIVE

Consider the simplest form of the linearized equation of motion of an undamped horizontal cantilevered pipe conveying fluid,

$$EI\frac{\partial^4 w}{\partial x^4} + MU^2\frac{\partial^2 w}{\partial x^2} + 2MU\frac{\partial^2 w}{\partial x \partial t} + (M+m)\frac{\partial^2 w}{\partial t^2} = 0, \tag{1}$$

where x and t are the axial coordinate and time, respectively, EI is the flexural rigidity of the pipe, M is the mass of fluid per unit length, flowing from the fixed end ($x = 0$) to the free one ($x = L$) with a steady flow velocity U, m is the mass of the pipe per unit length, and w is the lateral deflection of the pipe [see, e.g. Païdoussis (1998)]. Thus, for the present, we consider the pipe discharging rather than aspirating fluid. The first term in equation (1) is the flexural restoring force. Upon recalling that $\partial^2 w/\partial x^2 \sim 1/R$, where R is the local radius of curvature, it is obvious that the second term is associated with centrifugal forces as the fluid flows in curved portions of the pipe. Similarly, the third term is recognized as being associated with the Coriolis acceleration, and the last term represents inertial effects.

The dynamics of the system is well known for the case of $U > 0$. For sufficiently small U, the dynamics is dominated by the Coriolis force $2MU(\partial^2 w/\partial x \partial t)$, and the system is subjected to flow-induced damping. For sufficiently large U, however, the centrifugal force, $MU^2(\partial^2 w/\partial x^2)$, which may also be viewed as a compressive follower force, overcomes the Coriolis damping effect, and thus the system loses stability by single-mode flutter via a Hopf bifurcation.

Considering periodic motions of period T, it is shown (Benjamin 1961a; Païdoussis 1970, 1998) that the work done by the fluid on the pipe is equal to

$$\Delta W = - MU \int_0^T \left[\left(\frac{\partial w}{\partial t} \right)_L^2 + U \left(\frac{\partial w}{\partial t} \right)_L \left(\frac{\partial w}{\partial x} \right)_L \right] \mathrm{d}t \neq 0, \tag{2}$$

where $(\partial w/\partial t)_L$ and $(\partial w/\partial x)_L$ are, respectively, the lateral velocity and slope of the free end. For small $U > 0$, the first term dominates, and the work done is negative; hence, the pipe loses energy to the flowing fluid, and free pipe motions are damped. For high enough U, however, the second term dominates; if the slope and velocity of the free end have opposite signs over a period, $\overline{(\partial w/\partial x)_L(\partial w/\partial t)_L} < 0$, then the work done may be positive, and energy flows from the fluid (a source of unbounded energy) to the pipe, resulting in amplified oscillations. It should be noted that the aforementioned opposite-sign characteristic of the free-end slope and velocity corresponds to the "dragging, lagging" modal form of flutter, observed in experiments and commented upon by Bourrières (1939), Benjamin (1961b) and Gregory & Païdoussis (1966).

Consider next the situation with $U < 0$, i.e., the aspirating system. Exactly *the opposite* conclusions may be reached by consideration of equation (2): (i) in the course of free motions, the pipe absorbs energy from the fluid for all sufficiently small $|U|$ and is therefore subject to flutter; and (ii) for the higher $|U|$, the pipe loses energy to the fluid, and hence it is stabilized and its motions are damped. Consequently, the startling conclusion is reached that the system is unstable for infinitesimally small $|U|$ or, if dissipation is taken into account, for *very* small $|U|$.

These findings are confirmed by the full-fledged analysis of Païdoussis & Luu (1985) for this very system, and also for systems modelling a vertical pipe of the type used for ocean mining. Ocean mining is basically the 'vacuuming' of minerals, notably of manganese nodules, which lie on the floor of the ocean, e.g. in the Northeast Pacific, at depths of the order of 5 km. The system involves a very long "vacuum hose", with a massive "vacuum head" which walks along the ocean floor, scouring and sucking up nodule-rich sea water. It is clear that, the moment the bottom head loses contact with the sea floor, the system becomes a cantilevered pipe aspirating fluid and hence, by the foregoing arguments, subject to flutter.

Clearly, if this were true, ocean mining could be in trouble. For a typical system modelling an ocean mining pipe, including dissipative effects, the critical flow velocity for up-flow was found to be $|U| \sim 0{\cdot}2$ m/s (Païdoussis & Luu 1985), too small for comfort!

Hence, it was decided that time was ripe for experimental verification of Païdoussis & Luu's theoretical finding.

Some early experiments at the Chalk River Nuclear Laboratories in the mid-1960s had been inconclusive (Païdoussis 1998); hence, a new apparatus was built at McGill in 1986, shown in Figure 1(a). The entire elastomer pipe, hung vertically, was immersed in water in a steel tank; water was supplied at the top of the tank, and was forced up the hanging pipe and out of the vessel. Compressed air could additionally be supplied at the top of the tank to achieve higher flows over a limited time period, but also to conduct experiments entirely with air up-flow. Several experiments were conducted, with thicker pipes to postpone the shell-type buckling collapse (flattening) of the pipe,† and some with different-shaped inlet forms added, but the system remained unnervingly stable. The experiment was discontinued when, with ever increasing air pressure to force higher water flow up the pipe, the rubber hose leading the water to the drain burst free of its clamp, spraying water all over the laboratory and the instrumentation nearby, *and* giving the author an unwelcome cold shower! At that point, the author was certain that something was amiss with the theory; for one thing, the flow into the pipe is not exactly tangential, thus not replicating in reverse the outpouring jet in the case of down-flow. However, these negative results were not published, precisely because they were negative and not understood—which is why the tale is worth telling.

It was in 1995, during a visit by the author to Cambridge and while recounting this paradoxial behaviour that Dr D. J. Maull recalled reading "something similar" in Richard Feynman's biography (Gleick 1992).‡ It turns out that in 1939 or 1940, Feynman's and most other physicists' tea-time conversation at Princeton and the Institute for Advanced Study was dominated by this problem: if a simple S-shaped lawn sprinkler were made to suck up water instead of spewing it out [Figure 1(c)], would it rotate backwards or in the same way as for normal operation? (This problem was tied to the issue of reversibility of atomic processes!) Feynman could apparently argue convincingly either way.

Eventually, Feynman decided to do an experiment which, as shown in Figure 1(b), was remarkably similar to the author's. He immersed the lawn sprinkler into a glass jar filled with water, with an outlet connected to the sprinkler and a compressed air supply to force the water into the sprinkler and out. With increasing pressure and flow, the sprinkler refused to budge, up to the point where the glass jar exploded, spraying water all over. The result was that Feynman was banished from the laboratory henceforth!

Clearly, therefore, we have a paradox. Theory predicts that the aspirating pipe loses stability for infinitesimal (or very small) flow velocity, but experiments show the system to remain stable, at least to the maximum attainable flow prior to pipe collapse. Hence, reversing the flow direction in the experiments does not invert the stability behaviour of the pipe. Similarly, in Feynman's sprinkler, reversing the flow direction did not reverse (nor replicate) the direction of rotation.

3. RESOLUTION OF THE PARADOX AND NEW THEORY

Clearly, the flow field is entirely different in "forward" and "reverse" flow through the sprinkler. This is the key that finally led the author to the resolution of the conundrum, for both the sprinkler and the pipe problem. Consider the stationary aspirating sprinkler, and imagine a flared funnel, not connected to it, channelling the flow in, thus modelling the sink

† This collapse, due to viscous pressure drop in the internal fluid, relative to the external stagnant fluid, represents the ultimate limitation to the maximum flow that can be attained.

‡ It is with great sadness that this footnote is added. David Maull, a brilliant scholar and a good friend, passed away on 3 January 1999 poignantly, over the same weekend that this paper was being written.

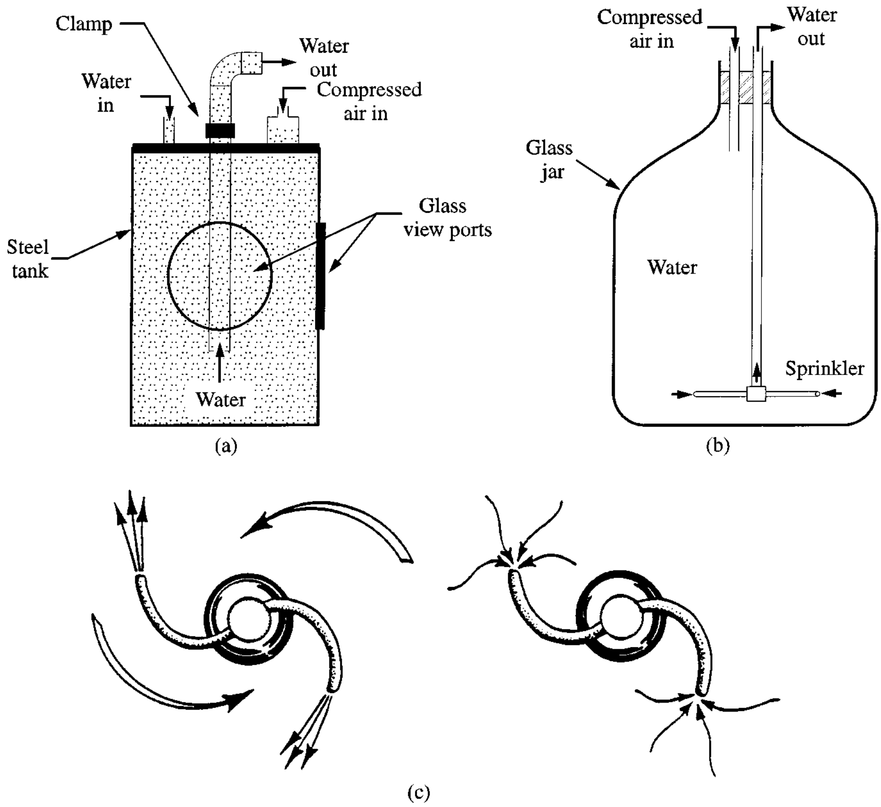

Figure 1. (a) New apparatus for forcing the fluid up the pipe in experiments by Païdoussis at McGill in 1980s; (b) Richard Feynman's apparatus for resolving the sprinkler problem at Princeton in late 1939 or 1940; (c) the sprinkler problem: which way does the sprinkler turn when aspirating fluid (Gleick 1992)?

flow. Neglecting gravity, the axial balance of forces in the funnel is given by (Païdoussis 1998)

$$\frac{\partial}{\partial x}\left[T + p_e A_e - p_i A_i - \rho_i (A_i U_i) U_i\right] = 0, \tag{3}$$

where now the internal flow velocity U_i is directed from the free towards the clamped end, and all quantities except ρ_i are functions of x; $p_e(x)$ and $p_i(x)$ are the external and internal pressures, and $A_e(x)$ and $A_i(x)$ the external and internal funnel cross-sectional areas. The tension T is taken up by the imaginary funnel supports and may be ignored. Also, this expression may be simplified by taking $A_e \simeq A_i = A$, and by writing $U_i = U$ and $p_i - p_e = p$, and recalling that $\rho_i A_i U_i = MU = \text{const.}$ Then, integrating from $x = \infty$, where $p \to 0$ and $U \to 0$, to $x = L$, the inlet of the sprinkler, we obtain $(pA)_L = -(MU^2)_L$. Hence, since MU^2 is the same for all $x < L$, one can write

$$\bar{p}A = -MU^2, \tag{4}$$

which clearly shows that at the sprinkler inlet, and hence throughout, there is a *suction* or *negative pressurization*, $\bar{p} = -\rho U^2 \equiv -MU^2/A$. Its effect is profound, as may be seen in Figure 2. The negative pressurization produces a lateral force $\bar{p}A/R = -MU^2/R$, R being the radius of curvature, which totally cancels the centrifugal force MU^2; hence, the sprinkler

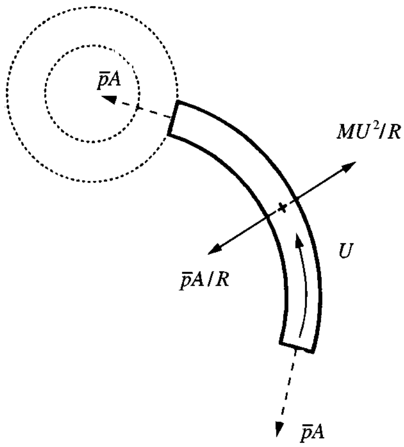

Figure 2. Negative pressurization and centrifugal force resultants on one arm of the sprinkler, effectively cancelling each other out (Païdoussis 1998).

remains inert! An alternative demonstration of this result may be made by control volume considerations and the fact that inlet and outlet vorticity is zero. Of course, these arguments do not hold once some rotation of the sprinkler takes place, but may be considered to be correct to zero order.

The same applies to the pipe problem. Unlike the case of discharging fluid where the pressure at the free end (above the ambient) is zero, for the aspirating pipe there is a suction at the free end, equal to $-\rho U^2$,[§] and hence a negative pressurization equal to that, throughout the pipe.[¶] Therefore, a term equal to $\bar{p}A(\partial^2 w/\partial x^2)$ must generally be added to the equation of motion, where $\bar{p}$ is the pressurization (depressurization), and equation (1) becomes

$$EI\frac{\partial^2 w}{\partial x^4} + (\bar{p}A + MU^2)\frac{\partial^2 w}{\partial x^2} + 2MU\frac{\partial^2 w}{\partial x \partial t} + (M+m)\frac{\partial^2 w}{\partial t^2} = 0. \tag{5}$$

For "down-flow", $U > 0$, $\bar{p} = 0$. For up-flow, $U < 0$, however, and as shown in the foregoing, equation (4) may be used for $\bar{p}A$, at least to zero order, and hence the centrifugal force in equation (5) vanishes. With no centrifugal (or follower) force, flutter cannot occur in the system!

4. EXPERIMENTAL VERIFICATION

In view of the past history of this system, and despite all the misadventures that have befallen the author and Feynman earlier still, it was imperative to have experimental verification that flutter does not occur.

Because of the problem of shell-type collapse of flexible pipes as the suction flow is increased, a new approach was taken in which the *principle* underlying the mechanism leading to flutter would be tested, rather than the phenomenon itself. Namely, it was

§ More generally equal to $-\rho U U_j$ in the case of a flared inlet segment of the pipe, where U_j is the flow velocity in the flared segment.

¶ This is *not* an inviscid flow result. Considering friction-related pressure drop, one obtains $\partial(pA - T)/\partial x = 0$; hence, $pA - T = \text{const}$ throughout the pipe. Moreover, $T_L = 0$, while $(pA)_L = -(MU^2)_L$; hence, $pA - T = (pA)_L = \bar{p}A$ throughout.

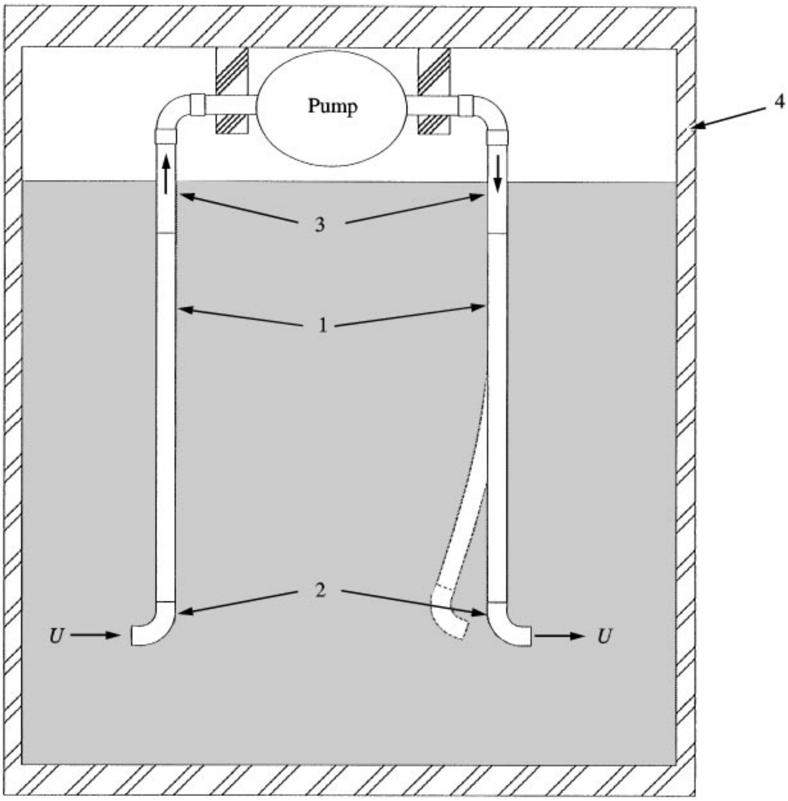

Figure 3. Schematic of the apparatus used to test the nonoccurrence of flutter in a pipe aspirating fluid. 1, identical, straight elastomer pipes; 2, identical rigid plastic elbows; 3, rigid metal piping, rigidly supported; 4, water-filled glass-walled basin in which the pipes were immersed. ——, shape of elastomer pipes for $U = 0$; – – –, shape for higher U. In the case of the aspirating pipe, the two shapes are coincident.

decided to test whether a centrifugal force does or does not arise with aspirating flow, as it does with discharging flow.

The apparatus constructed is shown in Figure 3. Two flexible straight elastomer pipes were fitted with light but rigid plastic elbows at their ends and hung as cantilevers in a water-filled basin. The clamped ends of the pipes were interconnected via a pump, such that one was aspirating flow and the other discharging the same flow.

Once the pump was started, the pipe discharging fluid deformed in reaction to the emerging jet, as expected. The aspirating pipe, however, after a starting transient returned to its original, no-flow configuration and thereafter remained limply straight. Therefore, it is now clear that aspirating pipes cannot aspire to flutter!

5. CONCLUSION

It has been shown that the earlier theory for aspirating pipes, which predicts that they are subject to flutter at very small or infinitesimal flow velocities is false. Basically, the flow entering the free end does not resemble a reverse jet, but rather a sink flow. Consequently, there exists a suction at inlet and hence throughout the pipe, cancelling out the centrifugal force which is at the core of the mechanism causing flutter—at least to first order. It is possible that this cancellation is not complete to higher-order approximations, and hence flutter cannot be excluded at high flow rates.

At the same time, the conundrum of "Feynman's sprinkler problem" has also been resolved. The two problems of the pipe and the sprinkler have been found to be analogous.

An experiment demonstrated the effective disappearance of the centrifugal force in an aspirating system, thereby lending support to the new theory.

Some aspects of the subject of this paper are discussed more amply in a recently published book (Païdoussis 1998), as is the resolution of other "paradoxes" which may be of interest.

ACKNOWLEDGEMENTS

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References

Benjamin, T. B. 1961a Dynamics of a system of articulated pipes conveying fluid. I. Theory. *Proceedings of the Royal Society* (*London*) *A* **261,** 457–486.

Benjamin, T. B. 1961b Dynamics of a system of articulated pipes conveying fluid. II. Experiments. *Proceedings of the Royal Society* (*London*) A **261,** 487–499.

Bourrières, F. -J. 1939 Sur un phénomène d'oscillation auto-entretenue en mécanique des fluides réels. *Publications Scientifiques et Techniques du Ministère de l'Air*, No. 147.

Dupuis, C. & Rousselet, J. 1991 Discussion to papers published earlier in JFS. *Journal of Fluids and Structures* **5,** 597–600.

Gleick, J. 1992 *Genius*. New York: Pantheon Books.

Gregory, R. W. & Païdoussis, M. P. 1966 Unstable oscillation of tubular cantilevers conveying fluid. II. Experiments. *Proceedings of the Royal Society* (*London*) A **293,** 528–542.

Holmes, P. J. 1978 Pipes supported at both ends cannot flutter. *Journal of Applied Mechanics* **45,** 619–622.

Païdoussis, M. P. 1970 Dynamics of tubular cantilevers conveying fluid. *I. Mech. E. Journal of Mechanical Engineering Science* **12,** 85–103.

Païdoussis, M. P. 1991 A note added to the discussion by Dupuis & Rousselet. *Journal of Fluids and Structures* **5,** 600.

Païdoussis, M. P. 1998 *Fluid–Structure Interactions*: *Slender Structures and Axial Flow*. London: Academic Press.

Païdoussis, M. P. & Luu, T. P. 1985 Dynamics of a pipe aspirating fluid, such as might be used in ocean mining. *ASME Journal of Energy Resources Technology* **107,** 250–255.